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Distributed Control of the Power Supply-Demand Balance

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***Abstract*—This paper aims to achieve a balance of power in a group of prosumers, based on a price mechanism, i.e. to steer the difference between the total production and consumption of power to zero. We first set the information network topology such that the prosumers exchange price (power) information with their neighbors according to a choosen information network topology. Based on the exchanged information and the prosumers own measured power demand, each prosumer uses a local control strategy to turn on and off its power generator to cooperatively achieve the global balance. More specifically, the local control strategy results from a distributed model predictive control method based on dual decomposition and sub-gradient iterations. The method achieves a unique dynamic price signal for each prosumer. Simulation results with realistic data validate the method.**

***Index Terms*—Distributed decision-making, distributed MPC, energy management, intelligent networks, modeling, optimal grid control, power distribution planning, price mechanism**

I. Introduction

In future power networks, large scale introduction of micro Combined Heat and Power (μ-CHP) systems are expected. A μ-CHP unit, in a household, produces heat that can be used for private consumption, and power that can be injected in the power network. Since both heat and power can be used, the overall energy efficiency of the μ-CHP is 1.24 times higher compared to a traditional power plant in combination with a gas-fired boiler [1]. The market potential is considered high [2], and the μ-CHP systems fueled on gas are of particular interest in countries like the Netherlands where the gas grid is dense.

The power output of the μ-CHP can be controlled, even though it is subject to several operational constraints. An overview of control strategies for the μ-CHP is given in [3]. In [1] a Model Predictive Control (MPC) approach for the modeling of a μ-CHP unit with demand response is presented. However, the power production from the μ-CHP influences the power balance in the network. It is therefore of importance to consider how a large number of μ-CHPs can influence the real time balance of power in the network.

Within the setting of Smart Grids, the households (agents) have the potential to contribute to the balance of the system [4]. We therefore foresee a shift towards a situation where a large number of smaller agents have more market power if they can coordinate their decisions. The motive can be both economical and environmental, as we assume a better price for the power when the resources are used more efficiently, and it is better for the environment when the energy losses are reduced.

In this paper, we study a possible optimal control scheme based on a price mechanism, for a network of μ-CHPs. We set two requirements for the scheme. First, in a large scale power network there are power losses in the transportation lines. Therefore, the objective of the control scheme is to map local power production to local consumption. Secondly, the control scheme has to scale well. We therefore look for a distributed approach, meaning that each agent make their own decisions whether their μ-CHP should be on or off based on local information.

It is widely agreed that a centralized solution scheme for the optimal control problem is too time consuming, because of the computational complexity [5]. Therefore, efforts have been made suggesting scalable control methods in the Smart Grid setting. In the literature, it has been stated that common for the strategies used for device control at household level, is that there is one decision making agent present, see [6]. In the PowerMatcher [7] for example, an agent for each device broadcasts a bidding curve for his willingness to pay for electricity. One agent at the top of a hierarchical structure, then determines the equilibrium price. Since the prices are the same everywhere in the network, there is no preferred location for the production in the network. In [6] a methodology combining forecasts, planning and real time control, that is capable of distinguishing position in the network, is described. However, the planning is centralized.

To avoid a centralized structure, we here propose an information network where each agent has local (imbalance) information about the system when they make their decisions. In a large network, the distance between suppliers and consumers are playing a role. An agent is not exchanging imbalance information with everybody, but bargains directly with a subset of all agents in the network according to the information structure. The idea is that the system as a total reaches a balance as if it could bargain with all agents directly, but now there is an ordering by information distance to neighbors from who an agent buys his power from. If the power is available at a direct neighbor, the agent will buy from this neighbor, and the power exchange is done locally. In the case that an agent needs to buy from a neighbor that is not a direct neighbor, he

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must exchange information through his neighbors' neighbor connections until an agent wants to sell.

The agents in the information network is an isolated subset of the agents in the power network. The goal is to minimize the difference between power demand and production in the information network by locally determining the off (on) state of the μ-CHPs. We assume that external parties are responsible for the overall power balance in the network, i.e. the information network is contributing to the overall balancing. We do not consider here that the demand will be influenced, i.e. the comfort levels of the agents are not altered.

Our proposed model can be used with an iterative method so that the agents decisions are made in a completely distributed way. Such a control strategy using a price mechanism is described in [8], [9]. This strategy, based on dual decomposition, is applied to control a formation of vehicles [8]. By exchanging only prices, the vehicles hold the desired position. This is also an attractive idea for control of decentralized power generation. We adopt these methods and apply them to the power supply-demand balance setting. Very preliminary results of our work were presented in [10] and [11]. In [10], dynamic price mechanisms were introduced to coordinate decisions. However, the consideration of technical constraints from the μ-CHP was lacking. Due to the constraints, MPC is a useful technique to solve the optimal control problem, e.g. [12]. It is also a framework where predictions and forecasts of the agents behavior is naturally included. In [11] preliminary results using a distributed MPC method [13] was presented. Agent based MPC to load-frequency control is presented in [14].

The main contribution here is the embedding of the μ-CHPs in the power network using a fully distributed MPC setting [13] together with our information sharing model. The method includes forecasts, real time optimal control and distributed decision making to achieve power balance. We examine practical control considerations due to the on (off) restriction of the μ-CHP. Two different strategies for inclusion of the on (off) characteristics of the μ-CHPs are presented. One strategy uses a quadratic program to solve the problem, while the other uses a mixed integer quadratic program to find the solution. The methods are tested with realistic power demand patterns from different types of households, and the scalability to a network of 1000 agents is considered.

The rest of this paper is organized as follows. Section II develops the dynamic models of the agents in the network, defines the μ-CHP constraints, and defines the control objective. Section III reviews an existing distributed MPC technique based on dual-decomposition and sub-gradient iterations. Section IV focuses on the technical details for explicitly including the off-on and range constraints from the μ-CHP to our model. One convex and one non-convex formulation is given. The simulation results, which verify the proper working of the method, are presented in Section V. Here realistic power demand patterns are used. Finally, Section VI discusses the results in this paper.

II. Modeling

In this Section, we give a brief description of our network model. The model is designed to be used for the coordination of power production and consumption in a multi-producer multi-consumer Smart Grid. The goal of the agents is to match the local power consumption and production, in order to avoid transport losses in the network, see Section II-C.

A. System description

The system consists of n agents (prosumers), which represent for example households with a μ-CHP or larger prosumers such as a hospital with a CHP. At the discrete time-step k agent i has a power demand $d_i(k) \in \mathbb{R}_+$ and a power production $p_i(k) \in \mathbb{R}_-$. The demand is an external signal that is only measured at each time step k, which means that we are not altering the comfort level of the agents. It is the production that can be adjusted by the agent. Therefore, the decision to be made at each agent is how much to ramp up (down) the power production $u_i(k)$, where

$$p_i(k) = p_i(k-1) + u_i(k). \tag{1}$$

We call the difference between power production $p_i(k)$ and demand $d_i(k)$at an agent i, the real imbalance $\tilde{x}_i(k)$ at agent i. Since we define the demand to be positive and the production to be negative, the agents' imbalance is defined by

$$\tilde{x}_i(k) = d_i(k) + p_i(k),\ \forall k \geq 0, \tag{2}$$

and the dynamical behavior is given by

$$\tilde{x}_i(k+1) = \tilde{x}_i(k) + u_i(k) + w_i(k),\ \forall k \geq 1, \tag{3}$$

where we have introduced $w_i(k) = d_i(k) - d_i(k-1)$ to represent the change in power demand at agent i.

In order for an agent to contribute to the local balancing of power by selling or buying power from neighbors, the agent requires some information about the overall power situation in the network.

However, to avoid a centralized structure we introduce the state $x_i(k)$ which represents information about the imbalance of agent i, and depends also on information about imbalance of neighbouring agents. We introduce a virtual information network, so that each agent has local information about the system when making the decision. The topology of the information network specifies which subset of agents an agent i exchanges information with. Agent i's set of information neighbors N_i is given by

$$N_i \subseteq \{1, \ldots, n\} \backslash \{i\}, \tag{4}$$

where the agent itself is excluded.

We include the chosen information topology in our dynamic model by adjusting information weights $A_{ij},\ i, j = 1, \ldots, n$ in the coupling between the agents' notion of imbalance in the system. The model for the *imbalance information* $x_i(k)$ at agent i is given by

$$x_i(k+1) = A_{ii}x_i(k) + \sum_{j \in N_i} A_{ij}x_j(k) + u_i(k) + w_i(k), \tag{5}$$

for all $i = 1, \ldots, n$, where A_{ii} weighs the power imbalance information of agent i itself, and A_{ij} weighs the information received from neighbors $j \in N_i$. We choose the initial value of $x_i(0)$ to be the real physical power imbalance of agent i

at the initial time, i.e. $x_i(0) = d_i(0) + p_i(0)$ and $w_i(0) = 0$. As time evolves, demand changes are spread though the network through the neighboring agents N_i. In this way close-by information-exchange agents can react faster to a change in demand $w_i(k)$ than information exchange agents further away. In Fig. 1, the solid lines represent one possible information network where each agent has one information neighbor $N_i = \{i-1\}$, and the self loops represent that agents take their own imbalance information into account as well.

In addition to the information network, all agents $i = 1, \ldots, n$ are physically connected to the power grid, which is illustrated by the stippled lines in Fig. 1. This means that the power demand and production at each agent will affect the overall power imbalance in the system $\sum_{i=1}^{n} \tilde{x}_i(k)$. We relate the imbalance information to the physical power imbalance by proper choices of the weights in the information matrix A.

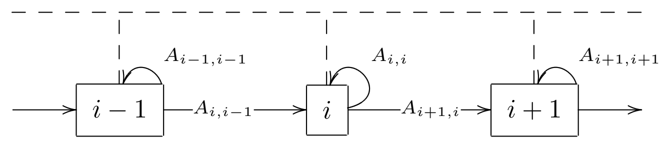


Fig. 1. Three agents which are both connected in the power grid, represented by stippled lines, and in one possible information network, represented by solid lines.

Notice that in (5) the *physical* imbalance enters the system at each agent i through change in production $u_i(k)$ and change in demand $w_i(k)$, and recall that $x_i(k+1)$ is the *information* about imbalance. Hence, each agent i has a state equation (5) also involving imbalance information from neighboring agents $j \in N_i$. The information topology of the network is specified by the *information matrix* $A \in \mathbb{R}^{n \times n}$, where the weights A_{ii}, A_{ij} in (5) are the elements of A. We impose four restrictions for how to choose these weights:

- R1 $A_{ij} \neq 0$ if and only if information is exchanged from agent j to agent i.
- R2 All weights are non-negative: $A_{ij} \geq 0,\ i, j = 1, \ldots, n$.
- R3 All columns sum up equal to one: $\sum_{i=1}^{n} A_{ij} = 1,\ j = 1, \ldots, n$.
- R4 The graph corresponding to information matrix A is strongly connected [15]. Loosely said: there is a path in the communication graph from any agent to any agent.

Requirement R1 specifies the information network topology. The topology is a design choice, however, more neighbors means that more information has to be exchanged in the network. Requirements R2 and R3 ensure that A is a stochastic matrix, so that the information is conserved. Finally, R4 ensures that the uncontrolled system is stable, since A is a stochastic matrix it follows from the Perron-Frobenius Theorem that the spectral radius is one, see [16]. Further, notice that R1-R4 still leave quite some freedom to choose the weights in A. The topology of Fig. 1 can be captured, but also other possible topologies with information exchange between more neighbors can be captured.

With the above requirements on the information weights A_{ij} and initial conditions $x_i(0) = \tilde{x}_i(0) = d_i(0) + p_i(0)$, substituting (3) and (5) into (6) results in the relation

$$\sum_{i=1}^{n} x_i(k) = \sum_{i=1}^{n} \tilde{x}_i(k),\ \forall\ k \geq 0, \tag{6}$$

which means that the total imbalance information in the network is equal to the total power imbalance in the network even though $x_i(k) \neq \tilde{x}_i(k)$ at an agent level.

Remark 1. *The current power network can be captured in the information matrix A, as information neighbors are physically virtual neighbors. By choosing physical nearby neighbors (in the power network) as information neighbors, local production for local demand can be stimulated. This is because the agents that receive the information about a change in demand the fastest, will be able to react to this change first. Consequently transportation losses in the power grid are avoided.*

We assume that the overall power shortage (excess) in the system is imported from (exported to) an external network. We do not include this explicitly in our model, but in Section II-C, we will formulate our objective with the aim to minimize this exchange.

B. Physical constraints from the μ-CHP

The μ-CHPs have physical restrictions that constrain the control input $u_i(k)$ that can be implemented at each agent. The change in power production $u_i(k)$ is related to the production $p_i(k)$ through a dynamic equation. Depending on its implementation, the change in production $u_i(k)$ can be found in $p_i(k)$ at the same time-step k or later. We choose to model this relation as in Eq. (1).

We include a range constraint that reflects that there is a maximum power $p_{i,\text{max}} > 0$ and a minimum power $p_{i,\text{max}} > p_{i,\text{min}} \geq 0$ that the μ-CHP can deliver. Here $p_{i,\text{max}}, p_{i,\text{min}} \in \mathbb{R}_+$ are constant scalars, but may have different values for each agent i depending on the size of the μ-CHP.

Ultimately we want the power output to be either zero or within a range. Hence, the production set is defined by

$$P_i(k) = \{p_i(k) | -p_{i,\text{max}} \leq p_i(k) \leq -p_{i,\text{min}}\} \cup \{0\}, \tag{7}$$

for all $k > 0$. This is a *non-convex* set when $p_{i,\text{min}} > 0$. In practice $p_{i,\text{min}}$ will always be strictly larger than zero because the μ-CHP can not produce infinity small amounts of power. Such non-convexities are a potential challenge when solving optimal control problems.

In addition, we include constraints that reflect the presence of a minimum run-time $T_{i,\text{on}}$ (minimum off-time $T_{i,\text{off}}$) where we require that the μ-CHP stays on (off) for at least $T_{i,\text{on}}(T_{i,\text{off}})$ time steps after the machine was turned on (off). This means that $\{0\}$ is excluded from the set (7) for $T_{i,\text{on}}$ time-steps after the μ-CHP is turned on. Similarly $\{p_i(k)| -p_{i,\text{max}} \leq p_i(k) \leq -p_{i,\text{min}}\}$ is excluded from the set (7) for $T_{i,\text{off}}$ time-steps after the μ-CHP is turned off.

Further details concerning the implementation, will be presented in Section IV.

C. Objective function

The objective is to map local power supply to local power demand. Therefore, our goal is to find the control input $u_i(k)$ for an agent i such that the imbalance $x_i(k)$ becomes zero for all agents $i = 1, \ldots, n$, given the influence from neighbors and physical constraints on the μ-CHP.

For a given imbalance $x_i(k)$ and change in production $u_i(k)$, we associate a cost $V_i(x_i(k), u_i(k))$ for each agent $i = 1, \ldots, n$. In particular we choose $V_i(x_i(k), u_i(k))$ at time k to be

$$V_i(x_i(k), u_i(k)) = R_{ii} x_i^2(k) + Q_{ii} u_i^2(k), \tag{8}$$

where the weights R_{ii} and Q_{ii} indicate the relative importance of each agent. If one wish to make imbalance of a large industry more important than e.g. a household, one can choose the corresponding weight higher.

The network cost $V(x(k), u(k))$ at time k is assumed to be the sum of the individual costs

$$V(x(k), u(k)) = \sum_{i=1}^{n} V_i(x_i(k), u_i(k)). \tag{9}$$

In our optimal control problem, the goal is to find the inputs $u_i(k)$, $i = 1, \ldots, n$, that minimizes the overall power exchange with large external power suppliers in the network over an infinite horizon such that the constraints hold. This objective becomes

$$\inf_{u} \lim_{K\to\infty} \sup \frac{1}{K} \sum_{k=0}^{K} V(x(k), u(k)), \tag{10}$$

which means that the network as a total wants to regulate the imbalance to zero at minimal production cost. Three reasons for introducing (8) and (9) in the form that we do are; 1: the function must have the minimum in zero in order to balance power supply and demand, 2: from an agent's perspective it is better to have a minimum per agent rather than squaring the sum of the agents, 3: from an optimal control and computational perspective quadratic cost functions are motivated by convexity and differentiability arguments.

Remark 2. *We have assumed that when the total imbalance $\sum_{i=1}^{n} x_i(k)$ is positive, the network has to import power from the external network, and when the total imbalance is negative the network is a provider of power to the external network.*

III. Optimal control for imbalance regulation

Here we define our optimal control problem and give the distributed Model Predictive Control (MPC) algorithm to find the change in productions $u_i(k)$, for $i = 1, \ldots, n$, in a completely distributed manner. First we introduce the centralized MPC to set the notation, and then we continue with the distributed MPC, in which the central MPC problem is decomposed into substantially smaller subproblems. Each subproblem is iteratively solved independent of each other and combined into a global solution.

A. Centralized MPC

Due to the constraints on the control input $u_i(k)$ and production $p_i(k)$, we choose to solve the optimal control problem in the MPC setting. The solutions will be sub-optimal compared to solving (10), but we ensure that the constraints are met for all agents i for all time k. MPC is known to be robust with respect to external disturbances, and it is a systematic approach to take both static and dynamic constraints into account [12]. The method is widely used in process industry [17].

One of the basic ideas of MPC is the inclusion of models to predict the future dynamics of the system over a finite time-horizon. In order to label the predictions over the horizon T, we introduce a new time variable $\tau = k, \ldots, k+T$. The value $\hat{x}_i(\tau)$ is predictions for imbalance $x_i(k)$ over the horizon. For the rest of the paper the hat notation indicates predictions of the variables. In the *centralized MPC* problem, a modified optimal control problem of (10) is solved at each time-step k over a *finite horizon* T. By solving

$$\min_{\hat{u}} \sum_{\tau=k}^{k+T} V(\hat{x}(\tau), \hat{u}(\tau)) \tag{11}$$

where $V(\cdot,\cdot)$ is the net quadratic cost (9), the centralized MPC solution is obtained. The minimisation problem (11) is subject to the following prediction models based on (5) and (1) for all $\tau = k, \ldots, k+T$ and all $i = 1, \ldots, n$,

$$\begin{aligned} \hat{x}_i(\tau+1) &= A_{ii}\hat{x}_i(\tau) + \sum_{j\in N_i} A_{ij}\hat{x}_j(\tau) \\ &\quad + B_{ii}\hat{u}_i(\tau) + \hat{w}_i(\tau), &(12)\\ \hat{p}_i(\tau) &= \hat{p}_i(\tau-1) + \hat{u}_i(\tau), &(13) \end{aligned}$$

where the set of neighbors N_i is given by (4). Different models for the change in demand $\hat{w}_i(k)$ can be included. This can be a forecast based on information from the agent or on historical data, or in the simplest case; the demand stays the same over the horizon.

The initial conditions and additional constraints are given by

$$\begin{aligned} \hat{x}_i(\tau)_{\tau=k} &= x_i(k), \\ \hat{p}_i(\tau)_{\tau=k} &= p_i(k), \\ \hat{x}_i(\tau) \in X_i, \;\; \hat{u}_i(\tau) &\in U_i, \\ \hat{w}_i(\tau) \in W_i, \;\; \hat{p}_i(\tau) &\in P_i(\tau), \end{aligned} \tag{14}$$

where the constraint set $P_i(\tau)$ is determined by the constants presented in Section II-B and X_i, U_i, W_i are convex sets, see [12],[17]. The change in demands $w_i(k)$ are assumed bounded $|w_i(k)| \leq w_{\max}$, as the houses have an upper consumption limit set by the network.

Notice that solving problem (11) involves the notion of future states $\hat{x}_j(\tau)$ of neighbors $j \in N_i$ which again depend on their neighbor connections. Problem (11) can only be solved if the controller can access the evolution of all states $\hat{x}_j(\tau)$, $j = 1, \ldots, n$ in the network. This is the central control problem, and computational problems are expected for large networks. Therefore, we will reformulate this problem as a distributed control problem where every agent in the network makes decisions only based on local information.

B. Distributed MPC

In [9] it has been shown that dynamic price mechanisms result from the dual decomposition method for distributed optimization of feedback systems. In [13] the method is combined with MPC, where the dual decomposition technique for MPC with sub-gradient iterations is explained.

We define

$$\hat{v}_i(\tau) = \sum_{j \in N_i} A_{ij} \hat{x}_j(\tau), \tag{15}$$

as the influence agent i expects to receive from its neighboring agents $j \in N_i$, where N_i is defined in (4). The prediction model corresponding to equation (5) then yields a *decoupled state equation*

$$\hat{x}_i(\tau+1) = A_{ii}\hat{x}_i(\tau) + \hat{v}_i(\tau) + B_{ii}\hat{u}_i(\tau) + \hat{w}_i(\tau), \tag{16}$$

with additional constraints (15) to problem (11). The guess $\hat{v}_i(\tau)$ will be calculated in the local optimization problem of agent i. This means that $\hat{v}_i(\tau)$ will be an extra optimization variable in the problem.

The dual decomposition technique (or Lagrange relaxation) requires the introduction of new optimization variables to the problem; the Lagrange multipliers

$$\hat{\lambda}(\tau) = [\hat{\lambda}_1(\tau), \dots, \hat{\lambda}_n(\tau)]' \in \mathbb{R}^n \tag{17}$$

which can be *interpreted as price* signals between neighboring agents [8].

We obtain a set of decoupled minimisation problems. Define

$$\begin{aligned} V^i_{\text{agent}} = \sum_{\tau=k}^{k+T} \Big(& V_i(\hat{x}_i(\tau), \hat{u}_i(\tau)) \\ & + \hat{\lambda}_i(\tau)\hat{v}_i(\tau) - \sum_{j \neq i} \hat{\lambda}_j(\tau) A_{ji} \hat{x}_i(\tau) \Big). \end{aligned} \tag{18}$$

so that the set of decoupled minimisation problems are given by

$$\min_{\hat{u}_i} V^i_{\text{agent}}, \tag{19}$$

$$\text{s.t. } (16), (13) \text{ and } (14) \text{ hold.} \tag{20}$$

Thus, only price information $\hat{\lambda}_j(\tau)$ from connected agents $j \in N_i$ is needed to solve the decoupled minimisation problems (19).

In order to make the algorithm fully distributed, sub-gradient iterations are included e.g. [13]. This can be done since the dual cost is concave in $\hat{\lambda}(\tau)$, even if the original problem (11) is not convex e.g. [18]. By including the sub-gradient iterations, Problem (11) is approximated. For all $\tau = k, \dots, k+T$ the sub-gradient iterations of the prices are updated according to

$$\hat{\lambda}_{i,r+1}(\tau) = \hat{\lambda}_{i,r}(\tau) + \gamma_{i,r}[\hat{v}_{i,r}(\tau) - \sum_{j \in N_i} A_{ij} \hat{x}_{j,r}(\tau)], \tag{21}$$

where r is the counter for the gradient iteration. In this way the price updates are also distributed, only depending on information from neighboring agents $j \in N_i$, and the gradient-steps $\gamma_{i,r}$ are chosen such that we converge to the optimum.

The algorithm solves the local problem (19) at each agent i iteratively with the gradient steps (21). The algorithm terminates when $\hat{v}_{i,r}(\tau) - \sum_{j \in N_i} A_{ij}\hat{x}_{j,r}(\tau)$ converges to zero, and hence (15) is met.

The original information structure is preserved, and as a bonus $\lambda(k)$ can be interpreted as a price reference [18] [8]. Thus, a *distributed MPC* is obtained.

By reformulating the centralized MPC problem to a distributed MPC problem, prices are introduced in order to coordinate the decisions in the network. It is a property of the method itself that both the decisions and the dynamic prices are determined iteratively. In this way, we achieve a mutual dependence of decisions and prices. Notice that in the centralized formulation only imbalance is communicated between the agents, but the controller has to have access to all information for all the agents to make the decisions. In the distributed formulation, both the imbalance and the price are communicated between neighbouring agents, and there is a local controller present at each agent making decisions only depending on local information.

Remark 3. *The interpretation of the Lagrangian multipliers as prices or shadow prices comes form the economics and game theory literature [18]. It is useful to work with price like concepts when dealing with allocation problems [19]. Each agent i bases their decision on maximising their "cost" (19) given the input prices λ_j for $j \in N_i$. The iterative adjustment of the price is just like a market equilibrium process where the price is adjusted to match supply and demand. However, the price is not the price for power in euros. In our model, we view the price as a weighing parameter for the distance to the equilibrium price for power. Indeed, when the network is in equilibrium, all agents will have a Lagrangian multiplier equal to zero.*

IV. Explicit inclusion of μ-CHP constraints

Here we explicitly include constraints from the μ-CHP, in the local sub-problems (19). We propose two distinct schemes for including the operational constraints given in Section II-B, i.e. that the μ-CHPs can not produce very small amounts of power. The first formulation (Problem QP) is convex, which makes it fast to solve with available optimization algorithms and the theory provided in Section III-B is valid. However, we will sometimes face infeasible solutions from the optimization algorithm because the gap between zero and $p_{i,\min}$ is not taken into account. In this case, a sub-optimal solution has to be implemented in the MPC time-step, i.e. either zero or $p_{i,\min}$. The second formulation (Problem MIQP), is closer to reality, as the optimization problem includes the logics of turning the μ-CHP on and off. If the solution exists it will always provide a solution that can be implemented on the μ-CHP, i.e. a feasible solution. As $p_i(k)$ can not take its values in a continuous interval, we are not guaranteed to find a solution. Here, this means that we expect to encounter situations where the solution $u_i(k)$ to be implemented oscillates between turn on (off) and stay off (on), and by implementing on or off in MPC time-step the network will produce slightly more or less power than what is optimal.

In both problem formulations, we will find a feasible solution to implement at the MPC time step, even though the optimization does not provide a feasible solution. This means that due to the MPC formulation we can always guarantee that the constraints are met, but due to the gap between zero and $p_{i,\min}$ the solution will be sub-optimal. Since "Problem QP" is the computationally cheapest, we are interested to see whether this problem formulation is useful compared to the "Problem MIQP" formulation. In addition, since the second problem is non-convex we are interested to see if the algorithm still provides solutions.

A. Solving a Quadratic Program (QP)

In this formulation the constraint set $P_i(\tau)$ for the local sub-problems (19) is a pre-specified real interval set that is reset at each time-step k. The optimization does not include the off switching during an optimization cycle. Unless further restrictions are known up front, the constrained set is defined for all agents $i = 1, \ldots, n$ over the horizon $\tau = k, \ldots, k+T$ by

$$P_i(\tau) = \{\hat{p}_i(\tau) | -p_{i,\max} \leq \hat{p}_i(\tau) \leq 0\}, \tag{22}$$

where the minus sign is explained in Section II-B. If the state of the μ-CHP has changed, we include additional on (off) restrictions in $P_i(\tau)$. The minimum off-time T_{off} after shut down, and minimum on-time T_{on} after start up is ensured by specifying $P_i(\tau)$ at each MPC starting time-step k, before the optimization is performed. We use the $t_i(k)$ counter to keep track of how long the μ-CHP has been off (on). Regions of the set (22) are excluded according to

$$P_i(\tau) = \{0\}, \tag{23}$$

for $\tau = k, \ldots, k + T_{i,\text{off}} - t_i(k)$ and according to

$$P_i(\tau) = \{\hat{p}_i(\tau) | -p_{i,\max} \leq \hat{p}_i(\tau) \leq -p_{i,\min}\}, \tag{24}$$

for $\tau = k, \ldots, k + T_{i,\text{on}} - t_i(k)$, where the counter $t_i(k)$ is set to zero when the μ-CHP changes state, which means that both $p(k+1) = 0$ and $p(k) \neq 0$. Similarly, $t_i(k)$ is set to zero when $p(k+1) \neq 0$ and $p(k) = 0$. We stress that production sets can only change after an optimization step is finished, i.e. the on (off) change can not be taken into account inside the local minimisation problem (19).

We always converge to the global optimum by iteratively solving the local minimisation problem (19) and sub-gradient step (21), see e.g. [13]. However, when $-p_{i,\min} \geq p_i(k) \geq 0$, in fact we can not conclude whether we should turn the machine on or off.

1) Obtaining a physical solution: An ad-hoc solution to the problem of the gap between zero and minimum physical production from the μ-CHP, is to choose a threshold $p_{i,\min} > 0$ for the implementation only. If an input $u_i(k)$ is found that would result in a production $p_i(k)$ in the interval smaller than what the μ-CHP can produce $-p_{i,\min} \leq p_i(k) \leq 0$ we have to round the input after the optimization such that $p_i(k) = 0$ or $p_i(k) = -p_{i,\min}$. In this way the notion of turning the μ-CHP on (off) is imposed outside the minimisation problem. This way the constraints are met in the distributed MPC algorithm, even if the solution is sub-optimal.

B. Solving a Mixed Integer Quadratic Program (MIQP)

In contrast to *Problem QP* we here introduce a mechanism that resets $t_{\text{on,(off)}}$ *inside* the optimization itself. Thus, we include the logics of turning the μ-CHP on (off). The production sets $P_i(\tau)$ are now non-convex.

We use a Mixed Integer Quadratic Program (MIQP) to solve the optimization problems at each time step k. This type of program that deals with binary variables, is described in [20]. Solving such a combinatorial problem can be time consuming compared to the Problem QP.

Similar to [1], we introduce a binary optimization variable, $\hat{r}(\tau)$, which indicates whether the μ-CHP is running or not.

$$\hat{r}(\tau) = \begin{cases} 1 & \text{if the } \mu\text{-CHP is on,} \\ 0 & \text{if the } \mu\text{-CHP is off.} \end{cases} \tag{25}$$

For correct operation we also need to know if the μ-CHP is turned on or off at a given time step. To keep track of this action we introduce action variables $\hat{a}(\tau)$ that is -1 when the μ-CHP is turned off, 0 when there is no changes, and 1 when the μ-CHP is turned on.

The equation describing the relation between the run state of the μ-CHP and the action taken at the given time-step is given by

$$\hat{a}(\tau) = \hat{r}(\tau+1) - \hat{r}(\tau). \tag{26}$$

To include the minimum on and off times we require that

$$\begin{aligned} \hat{r}_i(\tau) = 0, \quad \tau = k, \ldots, k + T_{i,\text{off}} - \hat{t}(\tau)_{i,\text{off}}, \\ \hat{r}_i(\tau) = 1, \quad \tau = k, \ldots, k + T_{i,\text{on}} - \hat{t}(\tau)_{i,\text{on}} \end{aligned} \tag{27}$$

i.e. the μ-CHP is off for $T_{i,\text{off}}$ time-steps and on for $T_{i,\text{on}}$ time-steps. The dynamics of the on and off counters are specified by letting $\hat{t}(\tau+1)_{i,\text{off}}$ be incremented by one when $\hat{r}_i(\tau) = 0$ and reset to zero when $\hat{r}_i(\tau) = 1$. Similarly, $\hat{t}(\tau+1)_{i,\text{on}}$ is incremented by one with $\hat{r}_i(\tau) = 0$ and reset to zero when $\hat{r}_i(\tau) = 1$.

When the time constraints (27) are not active $\hat{r}_i(\tau)$ can take any value in the set $\{0, 1\}$ and so allowing $\hat{p}_i(\tau)$ to take values in $P_i(\tau) = \{\hat{p}_i(\tau) | -p_{i,\max} \leq \hat{p}_i(\tau) \leq -p_{i,\min}\} \cup \{0\}$. This is modelled by the following constraint for all $\tau = k, \ldots, k+T$,

$$\hat{r}_i(\tau) \cdot p_{i,\min} \leq \hat{p}_i(\tau) \leq \hat{r}_i(\tau) \cdot p_{i,\max}, \tag{28}$$

where T is the prediction horizon.

A difference with *Problem QP* is that when we do find a solution, we are guaranteed that it is a feasible solution. However, as the problem is non-convex we will sometimes not converge in the iterations between the sub-gradient step for prices and the minimisation to find a unique control input.

Remark 4. *Even though the price gradient iterations (21) converge for non-convex problems, the combined problem of gradient iterations and minimisation of the local minimisation might not converge when $p_{i,\min} > 0$ in (7).*

1) Choosing a solution: We can expect the solution of the problem with the above constraints in some cases to oscillate between on and off. In the implementation we stop the sub-gradient-optimization iterations if the input starts to oscillate, and the on (off) state before the oscillation is implemented to the system. This way the constraints are met in the distributed MPC setting.

V. Simulation Results

Here we show the results from simulations combining our proposed model presented in Section II with the method reviewed in Section III. We implement the constraints both as presented in Section IV-A and in Section IV-B. In this section we take the agents in the network to be households. The solutions are found by a QP-solver and a MIQP-solver from GuRoBi version 4.6 [21] with python 2.5. For details about the implementation see [22].

A. Network Simulations

We perform simulations with realistic power demand patterns [23] provided by the Energy research Center of the Netherlands (ECN). Five different types of households are taken into consideration, and the demand patterns represent half a day in a November month. At this time of the year we can assume that the heat demand is high in the houses, in which case the heat production from the μ-CHPs is not wasted. The resolution of the demand patterns are one minute. Each house can have unique constraints on the capacity of the μ-CHP. However, in these simulations the network all agents has the same production capacity.

We are free to use any prediction model for the change in demand $\hat{w}_i(\tau)$, since the demand is an external signal, but an accurate forecast enables the controller to anticipate on the future behavior. For all simulations here, we assume that each household can exactly predict their change in demand patterns in the future, i.e. $\hat{w}(\tau) = w(\tau)$. Fig. 2 - 4 are generated using a network with circular information topology, given by information matrix

$$A = \begin{bmatrix} 0.6 & 0.2 & 0 & 0 & 0.2 \\ 0.2 & 0.6 & 0.2 & 0 & 0 \\ 0 & 0.2 & 0.6 & 0.2 & 0 \\ 0 & 0 & 0.2 & 0.6 & 0.2 \\ 0.2 & 0 & 0 & 0.2 & 0.6 \end{bmatrix} \tag{29}$$

Each agent weights their own imbalance with 0.6 and two neighbor imbalances with 0.2, i.e. the agents finds his own imbalance most important.

Based on simulation results, we use a prediction horizon of $T = 8$. It is a trade-off between computation-time and accuracy of the result. When the minimum off-time T_{off} (minimum on-time T_{on}) is shorter than the prediction horizon T, the MIQP solver becomes slow, as the combinatorial complexity rises. For all households $i = 1, \dots, 5$, the minimum production is $p_{i,\min} = 0.3$ kW and the maximum production is $p_{i,max} = 1$ kW, which are realistic values from a typical μ-CHP system. The minimum time off $T_{i,\text{off}} = 15$ min and the minimum on time $T_{\text{off}} = 15$ min are values abstracted from a μ-CHP present in our lab. We use gradient step size $\gamma_r = \frac{0.4}{r}$ in (21) at iteration number r, and the algorithm at one MPC time-step k terminates when $\Delta\lambda_i < 10$ for all $i = 1, \dots, n$ ($\Delta\lambda_i < 50$ in Table I).

Fig. 2 shows the net imbalance, demand and production for the network of five distinct household types using the distributed version of *Problem QP*. We see that the power production, dotted line, mirrors the demand, dashed line,

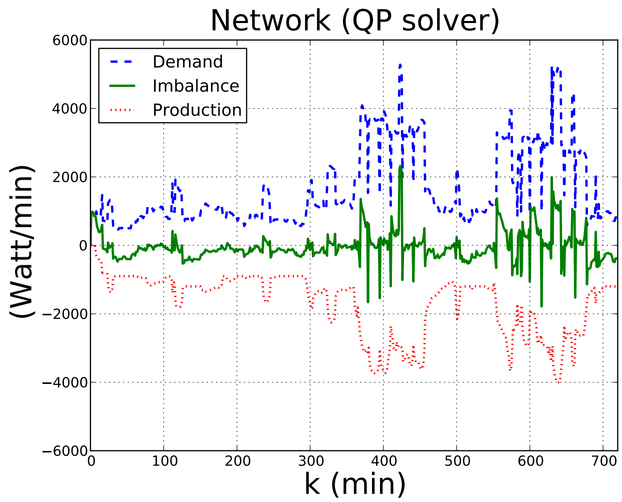
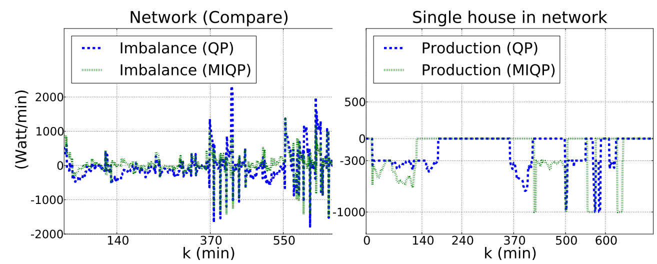


Fig. 2. The net imbalance $\sum_i x_i$, demand $\sum_i d_i$, and production $\sum_i p_i$ in a circular network of five households. The resolution is one minute, and the demand pattern for each household is unique. A Quadratic Program is used to solve Problem IV-A.



(a) (b)

Fig. 3. (a) compares the net imbalance of the QP and the MIQP formulation. (b) compares the production for a single household in the network.

nicely. Consequently, the imbalance shown in the solid line, is steered towards zero. Because of the good prediction models for change in demand, the network can anticipate the future situation.

We notice in Fig. 2, that the network is sometimes a net producer of power, see e.g. $k = 140$ where the imbalance is -475 kW. At other times the network is a net consumer, see e.g. $k = 370$ where the imbalance is 1152 kW. Without the μ-CHPs the network needs $7.20 \cdot 10^7$ kWh from the external line, and with the μ-CHPs the network needs $9.00 \cdot 10^5$ kWh from the external line. This is a reduction of 93.75 %. In addition, the network delivers $1.33 \cdot 10^7$ kWh in total in the same period.

For the network as a total, we obtain comparable results when we solve the distributed *Problem MIQP*. Fig. 3(a) is included to explicitly compare the solutions from the two formulations. Here the dashed line shows the imbalance in the network with the QP solver and the dotted line shows the imbalance in the network with the MIQP solver. The solution found by the MIQP solver seems to regulate the state better to zero, see e.g. $k = 550$ in Fig. 3(a).

There are clear differences at household level. In Fig. 3(b) we show the production patterns for one household, to see explicitly how a μ-CHP turns on and off. The dashed line shows the solution obtained from the QP solver and the dotted

TABLE I
INDICATION OF COMPUTATION TIME, NORMALIZED TO THE FIVE NODE CASE. NUMBERS ARE FROM THE QP-SOLVER.

Number of nodes	5	50	250	1000
Distributed MPC	1	2.01	3.38	3.38
Centralized MPC	1	5.83	27.27	-

line shows the solution obtained from the MIQP solver. If we focus on the interval $k = 500, \ldots, 600$, we see that using the QP solver the μ-CHP turns off during k = 190,...,361 and k = 422,...,500. While using the MIQP solver, the μ-CHP is turned off in the time slot k = 141,...,420. This means that even if the overall network performs comparably in the two different formulations, the individual division of who is selling and who is buying from neighbors differs. This is natural since MPC is sub-optimal in nature, and for the QP solution we do not know the effect of turning on or off the μ-CHP as this notion is enforced outside the optimization problem. Notice however also from Fig. 3(b) that when μ-CHP is on, the production is modulated in the range 0.3 kW till 1kW in both cases, which shows that the constraints are satisfied.

The overall cost (9) in the network for the different implementations, confirms the observation from Fig. 3(a) that the network performs slightly better with the MIQP formulation than with the QP formulation. The centralized MIQP cost $(8.40 \cdot 10^7)$ is 75% of the centralized QP cost $(1.15 \cdot 10^8)$. This is expected because we do not know the on-off effect in the QP formulation. The distributed implementation has a higher cost in both cases. In the QP case the cost $(1.24 \cdot 10^8)$ rises with 8.5% and in the MIQP the cost $(8.67 \cdot 10^7)$ rises with 3.2%. This has two reasons. Firstly, the on-off behavior of the μ-CHP makes the problem non-convex, and secondly, the algorithm is not performed until the price difference between two time steps is exactly zero.

Table I indicates that the distributed implementation scales well. For the distributed case we look at the average number of gradient iterations per node, while for the centralized case we look at computation time. The test is done for n = 5, 50, 250 and 1000 households, and the value for the five households case is set to one. The computations per household rises from 1 till 3.38 for 5 compared to 250 households in the distributed case, and the ratio stays at 3.38 for 1000 households. The computation time rise from 1 till 27.27 for 5 compared to 250 households in the centralized case, but for 1000 households the centralized problem can not be solved because the model was too large for the free GuRoBi licence.

B. Prices

The prices vary over the network, depending on the local imbalance. Fig. 4 shows the change in price for each household. We interpret negative values as prices higher than the equilibrium price and positive values as prices lower than the equilibrium price. The dotted line in Fig. 4 is the price pattern corresponding to the dotted line in Fig. 3(b). We observe that the value of the price in Fig. 4 rises just before the μ-CHP is turned on at $k = 420$ in Fig. 3(b). When the μ-CHP is switched on at k = 420 the value of the price immediately

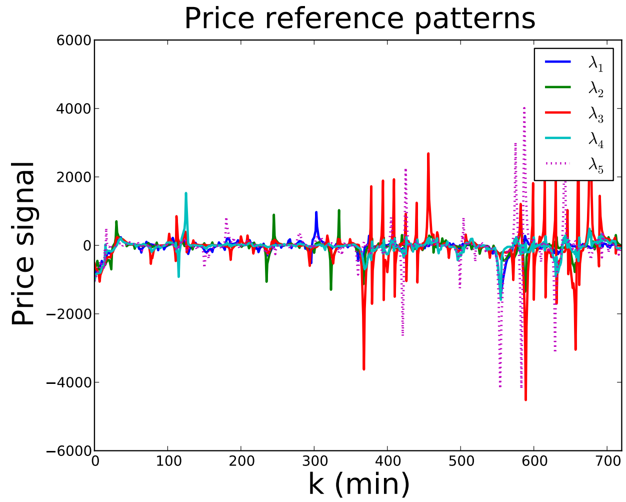


Fig. 4. The price references λ_i for each household type in the network.

decreases. Since the price rises when power shortage rises, this stimulates the device to be turned on when needed. When the problem (11) is convex, the prices in the network are such that no-one benefit from producing more of less power.

VI. DISCUSSION

In this paper, we have shown that distributed MPC via dual-decomposition and sub-gradient iterations is a suitable design approach for embedding distributed generation in the power network at end-user level. By coupling the agents dynamically through their notion of power imbalance information, and combining this model with the price mechanism the network as a total converge optimally to a balance of power supply and demand.

The model has the freedom to take into account different generator capacity on each agent, and the agents may be weighted with different relative importance in the cost-function and in the information network. Some rules for the weights of the information matrix A is given, and we explicitly showed how to include μ-CHP systems with on-off behavior in the distributed MPC method.

We include on-off behavior in two different versions in the simulations. One approach preserves the convexity in the optimization problem, in which case a fast algorithm can be used to find the solution and the theory reviewed in Section III is valid. The drawback is that the solution sometimes can not tell if the μ-CHP is on or off. The other approach explicitly takes into account the on-off behavior, but we are faced with more slow mixed integer algorithms to find the solutions, and the problem is non-convex.

Based on the simulation results in Section V, it can be concluded that both approaches steer the balance towards zero. The MIQP approach performs better with respect to the cost-function, while the QP approach is faster.

This research has raised many questions in need of further investigation. One question is how to best design our A matrix given the rules in Section II-A, since it affects both the steady state solution and the transient response of the system.

The current study has only examined flexibility in the power production, but when the price fluctuations are transparent for the end-user, we have reason to believe that we will also see flexibility in the demand. Demand response is treated in [24]. Further research to explore the modeling of the power demand side together with the distributed control approach, is currently under study.

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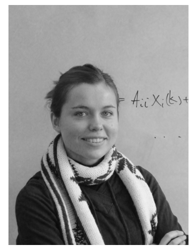

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